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DOUBLE–FRAMED LEVEL CUT OF FUZZY SOFT COMPOSITION RELATIONAL ASPECTS

[1,2]G.RETHNAREXLIN, [3*]G.SUBBIAH AND [4]V.NAGARAJAN

1. Research scholar, Reg.No: 18123152092017,Research Department of Mathematics,S.T.Hindu College, Nagercoil-629 002,Tamil Nadu, India.

2.Assistant Professor in Mathematics, Sarah Tucker College, Tirunelveli-627 007, Tamil Nadu, India.

3. *Associate Professor in Mathematics, Sri K.G.S. Arts College, Srivaikuntam-628 619, Tamil Nadu, India.

4.Assistant Professor in Mathematics, S.T.Hindu College, Nagercoil-629 002,Tamil Nadu, India.

* Corresponding author: E-mail Id: subbiahkgs@gmail.com

Affiliated to ManonmaniamSundaranar University, Abishekapatti, Tirunelveli-627 012, Tamil Nadu, India.

ABSTRACT: In this article, an attempt has been made to study level cut of double-framed fuzzy soft equivalence relation as well as its some properties such as equivalence class, union and intersection of equivalence relation. Some extension of intuitionistic fuzzysoft relations has been made into double-framed fuzzy softrelations.

Keywords: Soft set,fuzzy set, relation, symmetry, equivalence relation,union, intersection, equivalence class, double-framed fuzzy soft set.

1.INTRODUCTION: The fuzzy set was introduced by Zadeh [13] in 1965 where each element had a degree of membership. The intuitionistic fuzzy set on a universe was introduced by K.Atanassov [1] in 1986, as a generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. This concept has recently motivated new approach in several directions such as databases, medical diagnoses problems, decision making problem, topology, control theory and on so on. The concept of neutrosophic set handles indeterminate data whereas fuzzy theory and intuitionistic fuzzy set theory failed when the relations are indeterminate.Molodtsov introduced the concept of soft set theory in [10].Jun et.al [6] initiated to introduce double-framed soft sets and presented its applications in BCK/BCI algebras. Rosenfeld [11] applied the concept of fuzzy sets to the theory of groups and defined the concept of fuzzy subgroups of a group. Since then, many papers concerning various fuzzy algebraic structures have appeared in the literature [1, 2- 4, 7-9].In this article, an attempt has been made to study level cut of double-framed fuzzy soft equivalence relation as well as its some properties such as equivalence class, union and intersection of equivalence relation. Some extension of intuitionistic fuzzy soft relations has been made into double-framed fuzzy soft relations.

2. Preliminaries: In this section basic concept of double-framed fuzzy soft set has been reviewed.

2.1 Definition: Let 'U' be a Universe of discourse and 'A' be a set included in U. An element 'x' from U is defined as $A=\langle x,T_A(x),F_A(x)\rangle,\ x\in X$ where $\mathrm{T,F}:\mathrm{X}\to[0,1]$ and $0\le T_A(x)+F_A(x)\le 1,$ where T and F represent truth value and false value respectively.

2.2 Definition: Let 'X be a non-empty set and $A=\langle x,T_A(x),F_A(x)\rangle,\ \ B=\langle x,T_B(x),F_B(x)\rangle$ are double-framed sets.Then 'A' is a subset of B if $\forall\, x\in X,\ T_A(x)\ge T_B(x)$ and $F_A(x)\le F_B(x)$.

2.3Definition: Let 'X' be a non-empty set and $A=\langle x,T_A(x),\ F_A(x)\rangle,\ \ B=\langle x,T_B(x),F_B(x)\rangle$ are double-framed sets. Then

(i) $A\cup B=\{(x,\ \max\{T_A(x),\ T_B(x)\},\ \min\{F_A(x),\ F_B(x)\})/\,x\in X\}$

(ii) $A\cap B=\{(x,\ \min\{T_A(x),\ T_B(x)\},\ \max\{F_A(x),\ F_B(x)\})/\,x\in X\}$

(iii) $A=B\ if\ \ and\ only\ if\ \ \forall\, x\in X,\ T_A(x)=T_B(x)\ and\ F_A(x)=F_B(x).$

2.4 Definition: Let U be a non-empty set. Then by a fuzzy set on 'U' is meant a function A : U→[0,1]. A is called the membership function, A(x) is called the membership grade of x in A. we also write A = {(x, A(x)): x ∈ U}.

2.5 Example: Consider U = { a, b, c ,d } and A : U→[0,1] defined by A(a)=0, A(b)=0.7, A(c)=0.4, A(d)=1.

2.6 Definition: Let U be the initial universe set and E be the set of parameters. Let P(U) denote the power set of U. Consider a non-empty set A, A⊂ E. A pair (F, A) is called a soft set over U, where F : A→P(U).

2.7 Example: Suppose that U is the set of houses under consideration, say U={h_1, h_2, ..., h_5}.Let 'E' be the set of some attributes of such houses, say E = {e_1, e_2, ..., e_8}, where e_1,

e_2, …., e_8 stand for the attributes "expensive", "beautiful", "wooden", "cheap", "modern", "design", "in bad" and "repair" respectively.

In this case, to define a soft set means to point out expensive houses, beautiful housesand so on. For example, the soft set (F, A) that describes the "attractiveness of the houses" in the opinion of a buyer, say Thomas, may be defined like this: $A=\{e_1, e_2, e_3, e_4, e_5\}$; $F(e_1) = \{h_2, h_3, h_5\}$, $F(e_2) = \{h_2, h_4\}$, $F(e_3) = \{h_1\}$, $F(e_4) = U$, $F(e_5) = \{h_3, h_5\}$.

2.8 Definition: A collection (F, A) is called a double-framed fuzzy soft set if and only if $F : A \to P(U)$, when P(U) is the collection of all double-framed fuzzy soft set on the universal set 'U' and 'A' is a non-empty subset of the parameter set E.

2.9 Definition: Let (F, A) and (G, B) are two double-framed fuzzy soft setsover (U, E). Then the union of two double-framed fuzzy soft sets $C = A \cup B,\ \forall e \in C$

$$H(e) = \begin{cases} F(e),\ if\ \ e \in A - B \\ G(e),\ if\ \ e \in B - A \\ F(e) \cup G(e)\ if\ \ e \in A \cap B \\ and\ is\, writtenas\ (F,A) \cup (G,B) = (H,C) \end{cases}$$

Also the intersection of two double-framed fuzzy soft sets $C = A \cap B,\ \forall e \in C,$

$H(e) = F(c) \cap G(c)$ and is written as $(F,A) \cap (G,B) = (H,C).$

3.LEVEL CUT OF DOUBLE-FRAMED FUZZY SOFT RELATION

In this section, (s, λ)-cut of double-framedfuzzy soft set relation (DFSR) has been defined. Reflexivity, symmetricity and transitivity of double-framed fuzzy soft relation are defined. Equivalence of DFSR has been shown based on (s, λ)-cut. Later on, studied on equivalence class, union and intersection of equivalence DFSR's.

3.1 Definition: Let 'A' be a non-empty set. A double-framed fuzzy soft relation (DFSR) R on A is a double-framed fuzzy soft set (DFSS).

$$R = \{((a,b),\ T_A(a,b),\ F_A(a,b))/(a,b) \in A \times A\}$$

Where $T_A(a) = A \times A \to [0,1]\ and\ F_A(a) = A \times A \to [0,1]$ satisfying the condition $T_A(a,b) + F_A(a,b) \le 1,\ \forall (a,b) \in A \times A.$

3.2 Definition: A DFSR $R = \{((a,b),\ T_A(a,b),\ F_A(a,b))/(a,b) \in A \times A\}$ is said to reflexive if $T_A(a,a) = 1\ and\ F_A(a,a) = 0,\ \forall a \in A.$ Also, R is symmetric if

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$T_A(a,b)=T_A(b,a)$ *and* $F_A(a,b)=F_A(b,a), \forall (a,b)\in A\times A.$

3.3 Definition: If $R_1 = \{((a,b), T_1(a,b), F_1(a,b))/(a,b)\in A\times A\}$ and $R_2 = \{((a,b), T_2(a,b), F_2(a,b))/(a,b)\in A\times A\}$ be two DFSR's on A, then the composition denoted by $R_1 \circ R_2$ is defined by

$R_1 \circ R_2 = \{((a,b), T_1 \circ T_2(a,b), F_1 \circ F_2(a,b))/(a,b)\in A\times A\}$

Where $T_1 \circ T_2(a,b) = \sup\{\min\{T_1(a,c), T_2(c,b)\}\}, c\in A$

$F_1 \circ F_2(a,b) = \inf\{\max\{F_1(a,c), F_2(c,b)\}\}, c\in A.$

3.4 Definition: Let (s, λ) be the cut of double-framed fuzzy soft relation R. Then $\delta_{s,\lambda}(R) = \{(a,b)\in A\times A/T_R(a,b)\geq s, , F_R(a,b)\leq \lambda\}.$

3.5 Definition:A double-framed fuzzy soft relation R on A is called transitive if $R\circ R\subseteq R.$

3.6 Definition:A double-framedfuzzy soft relation R on A is called double-framed fuzzy soft equivalence relation (DFSER) if R is reflexive, symmetric and transitive.

3.7 Definition: For any double-framed fuzzy soft set $R = \{(x, T_A(a), \ F_A(a))/a\in A\}$ of set X, we define a (s, λ)-cut of A as the crisp set $\{a\in X/T_A(a)\geq s, \ F_A(a)\leq \lambda\}$of X and it is denoted by$\delta_{s,\lambda}(A)$.

3.8 Theorem: Let 'R' be a relation on a set X. Then 'A' is a DFS equivalence on X if and only if $\delta_{s,\lambda}(R)$is an equivalence relation on X, with $0\leq s,\lambda\leq 1$ and $s+\lambda=1$.

Proof:The condition is necessary.

We have $\delta_{s,\lambda}(R) = \{(a,b)\in X\times X/T_R(a,b)\geq s, \ F_R(a,b)\leq \lambda\}.$

Since, R is a DFS equivalence relation,so $T_A(a,a)=1\geq s$ *and* $F_A(a,a)=0\leq \lambda,$ *for all* $a\in X$.

Therefore, $(a,a)\in \delta_{s,\lambda}(R) \Rightarrow \delta_{s,\lambda}(R)$ is reflexive.

Now, let $(a,b)\in \delta_{s,\lambda}(R)$,then $T_R(a,b)\geq s$ *and* $F_R(a,b)\leq \lambda.$

But, R is DFS equivalence relation, so $T_R(b,a) = T_R(a,b)\geq s$ and $F_R(b,a) = F_R(a,b)\leq \lambda.$

Therefore, $(b,a)\in \delta_{s,\lambda}(R) \Rightarrow \delta_{s,\lambda}(R)$ is symmetric.Also, let $(a,b)\in \delta_{s,\lambda}(R)$ and $(b,c)\in \delta_{s,\lambda}(R)$, then $T_R(a,b)\geq s$ *and* $F_R(a,b)\leq \lambda$ *and* $T_R(b,c)\geq s$ *and* $F_R(b,c)\leq \lambda.$

Therefore, $\min\{T_R(a,b), T_R(b,c)\}\geq s$ and $\max\{F_R(a,b), F_R(b,c)\}\leq \lambda.$

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$\Rightarrow \max\{\min\{T_R(a,b), T_R(b,c)\}\} \geq s \Rightarrow (T_R \circ T_R)(b,c) \geq s$ and

$\Rightarrow \min\{\max\{F_R(a,b), F_R(b,c)\}\} \leq \lambda \Rightarrow (F_R \circ F_R)(b,c) \leq \lambda.$

But, R is DFS equivalence relation, so

$T_R(a,c) \geq (T_R \circ T_R)(a,c) \geq s$ and $F_R(a,c) \leq (F_R \circ F_R)(a,c) \leq \lambda.$

Therefore, $(a,c) \in \delta_{s,\lambda}(R) \Rightarrow \delta_{s,\lambda}(R)$ is transitive.

Therefore, $\delta_{s,\lambda}(R)$ is DFS equivalence relation of X.The condition is sufficient.

Suppose that, $\delta_{s,\lambda}(R)$ is an equivalence relation of X.

Taking s = 1 and λ = 0, we get $\delta_{s,\lambda}(R)$ is an equivalence and so a reflexive relation and so $(a,a) \in \delta_{s,\lambda}(R)\ \forall a \in X$. Therefore, Thus, $T_R(a,a) = 1\ and F_R(a,a) = 0$.Therefore, DFS relation R is reflexive.

For any $a,b \in X$, let $T_R(a,b) = s and F_R(a,b) = \lambda,\ then\, s+\lambda \leq 1$ and so by hypothesis $\delta_{s,\lambda}(R)$ is an equivalence and hence symmetric relation on X. Also, $(x,y) \in \delta_{s,\lambda}(R)$ so by symmetry $(y,x) \in \delta_{s,\lambda}(R)$. $T_R(b,a) \geq s = T_R(a,b)$ and $F_R(b,a) \leq \lambda = F_R(a,b)$.

Similarly, if $T_R(b,a) = \alpha\ and F_R(b,a) = \varphi, then\ (a,b) \in \delta_{s,\lambda}(R)$.

Now, $T_R(a,b) \geq \alpha = T_R(b,a)\, and\ F_R(a,b) \leq \phi = F_R(b,a)$

Here, $T_R(a,b) = T_R(b,a)\ and\ F_R(a,b) = F_R(b,a)$.

Therefore, DFS relation R is symmetric.

Again, let $a,b,c \in X$, then $\min\{T_R(a,c), T_R(c,b)\} = s$, $\max\{F_R(a,c), F_R(c,b)\} = \lambda$

Then $s \geq 0, \lambda < 1\ and\ s+\lambda \leq 1$.Therefore $\delta_{s,\lambda}(R)$ is an equivalence relation on X.

Therefore, $T_R(a,c) \geq s, T_R(c,b) \geq s\ and\ F_R(a,c) \leq \lambda, F_R(c,b) \leq \lambda.$

So, $\delta_{s,\lambda}(R)$ is an equivalence relation so by transitivity $(a,b) \in \delta_{s,\lambda}(R)$.

Then, $T_R(a,b) \geq s\, and\ F_R(a,b) \leq \lambda \Rightarrow T_R(a,b) \geq s = \min\{T_R(a,c), T_R(c,b)\}$ and

$F_R(a,b) \leq \lambda = \max\{F_R(a,c), F_R(c,b)\} \Rightarrow T_R(a,b) \geq \sup\{\min\{T_R(a,c), T_R(c,b)\}\}$ and

$F_R(a,b) \leq \inf\{\max\{F_R(a,c), F_R(c,b)\}\}.$

$\Rightarrow T_R(a,b) \geq (T_R \circ T_R)(a,b)$ and $F_R(a,b) \leq (F_R \circ F_R)(a,b)$

$\Rightarrow T_R \supseteq T_R \circ T_R\ and\ F_R \subseteq F_R \circ F_R.$

$\Rightarrow$DFS relation R is transitive.

Hence, DFS relation is an equivalence relation.

3.9 Definition: Let 'R' be a DFS equivalence on a set X and 'x' be an element of X. Then the DFS set defined by $xR = \{(a,\ xT_R(a),\ xF_R(a)) / a \in X\}$ where, $(xT_R)(a) = T_R(x,a)$ *and* $(xF_R)(a) = F_R(x,a)$, $\forall a \in X$ is called DFS equivalence class of x with respect to R.

3.10 Theorem: Let 'R' be a DFS equivalence relation on a set X and 'x' be any element of X. Then for $0 \le s, \lambda \le 3$ and $s + \lambda = 1$, $\delta_{s,\lambda}(xR) = [x]$ the equivalence class of x with the equivalence relation $\delta_{s,\lambda}(R)$ in X.

Proof: We have, $[x] = \{a \in X / (x,a) \in \delta_{s,\lambda}(R)\}$

$= \{a \in X / T_R(x,a) \ge s,\ F_R(x,a) \le \lambda\}$

$= \{a \in X / (xT_R)(a) \ge s,\ (xF_R)(a) \le \lambda\} = \delta_{s,\lambda}(R)$.

The proof is completed.

3.11 Theorem: Let 'R' be a DFS equivalence relation on a set X .Then $[x] = [y]$ if and only if $(x,y) \in \delta_{s,\lambda}(R)$ where $[x]$, $[y]$ are equivalence classes of 'x' and 'y' with respect to the equivalence relation $\delta_{s,\lambda}(R)$ in X for $0 \le s, \lambda \le 1$ and $s + \lambda = 1$

Proof: Let $[x] = [y]$.Then $\delta_{s,\lambda}(xR) = \delta_{s,\lambda}(yR)$

$\Rightarrow \{a \in X / (xT_R)(a) \ge s,\ \ (xF_R)(a) \le \lambda\}$

$= \{a \in X / (yT_R)(a) \ge s,\ (yF_R)(a) \le \lambda\}$.Let $a \in \delta_{s,\lambda}(xR) = \delta_{s,\lambda}(yR)$

$\Rightarrow (xT_R)(a) \ge s,\ \ (xF_R)(a) \le \lambda$ *and* $(yT_R)(a) \ge s,\ \ (yF_R)(a) \le \lambda$

$\Rightarrow (T_R)(x,a) \ge s,\ \ (F_R)(x,a) \le \lambda$ *and* $(T_R)(y,a) \ge s,\ \ (F_R)(y,a) \le \lambda$

$\Rightarrow \min\{T_R(x,a),\ T_R(y,a)\} \ge s,$

$max\{F_R(x,a), F_R(y,a)\} \le \lambda \Rightarrow \sup\{\min\{T_R(x,a),\ T_R(y,a)\}\} \ge s,$

$inf\{max\{F_R(x,a), F_R(y,a)\} \le \lambda\} \Rightarrow (T_R \circ T_R)(x,y) \ge s$ *and* $(F_R \circ F_R)(x,y) \le \lambda.$

$\Rightarrow (x,y) \in \delta_{s,\lambda}(R)$ .Conversely, let $(x,y) \in \delta_{s,\lambda}(xR)$,

$\Rightarrow T_R(x,y) \ge s$ *and* $F_R(x,y) \le \lambda \qquad \rightarrow (1)$

Let $x \in \delta_{s,\lambda}(xR)$.*Then* $(xT_R)(a) \ge s,\ \ (xF_R)(a) \le \lambda$

implies $(T_R)(x,a) \ge s,\ \ (F_R)(x,a) \le \lambda$

$\Rightarrow \min\{T_R(y,x), T_R(x,a)\} \geq s, \max\{F_R(y,x), F_R(x,a)\} \leq \lambda$ using (1)

$\Rightarrow \sup\{\min\{T_R(y,x), T_R(x,a)\}\} \geq s,$

$inf\{max\{F_R(y,x), F_R(x,a)\} \leq \lambda\} \Rightarrow (T_R \circ T_R)(y,a) \geq s$ *and* $(F_R \circ F_R)(y,a) \leq \lambda.$

$\Rightarrow (yT_R)(a) \geq s$ *and* $(yF_R)(a) \leq \lambda \Rightarrow (x,y) \in \delta_{s,\lambda}(yR) \Rightarrow \delta_{s,\lambda}(xR) \subseteq \delta_{s,\lambda}(yR)$

Similarly, $\delta_{s,\lambda}(yR) \subseteq \delta_{s,\lambda}(xR)$ .Hence, $\delta_{s,\lambda}(xR) = \delta_{s,\lambda}(yR) \Rightarrow [x]=[y]$.

Hence, the proof.

3.12 Theorem: The intersection of two DFS equivalence relations on a set is again a DFS equivalence relation on the set.

Proof: Let 'A' and 'B' be two DFS equivalence relations on a set X.

For any $0 \leq s, \lambda \leq 1$ and $s + \lambda = 1$

We have, $\delta_{s,\lambda}(A \cap B) = \delta_{s,\lambda}(A) \cap \delta_{s,\lambda}(B)$

We know that, $\delta_{s,\lambda}(A)$ *and* $\delta_{s,\lambda}(B)$ are equivalence relations on X and so $\delta_{s,\lambda}(A \cap B)$ is also an equivalence relation on X and $A \cap B$ is DFS relation on X.

3.13 Note: Union of two DFS equivalence relations on a set is not necessary a DFS equivalence relation on the set.

Let $X = \{x, y, z\}$ and 'A' and 'B' are two DFS sets on X,

Where, $T_A(x,x) = T_A(y,y) = T_A(z,z) = 1,$

$T_A(x,y) = T_A(y,x) = T_A(x,z) = T_A(z,x) = 0.1$ *and*

$T_A(y,z) = T_A(z,y) = 0.6$

$F_A(x,x) = F_A(y,y) = F_A(z,z) = 0, F_A(x,y) = F_A(y,x) = F_A(x,z) = F_A(z,x) = 0.2$ *and*

$F_A(y,z) = F_A(z,y) = 0.1.$

Again, $T_B(x,x) = T_B(y,y) = T_B(z,z) = 1,$

$T_B(x,y) = T_B(y,x) = T_B(y,z) = T_B(z,x) = 0.3$ *and* $T_B(x,z) = T_B(z,x) = 0.5$

$F_B(x,x) = F_B(y,y) = F_B(z,z) = 0.$ $F_B(x,y) = F_B(y,x) = F_B(y,z) = F_B(z,x) = 0.2$ *and*

$F_B(y,z) = F_B(z,y) = 0.1.$

Now, $((T_A \cup T_B) \circ (T_A \cup T_B))(x,y)$

$$=\sup\left\{\begin{array}{c}\min\{(T_A \cup T_B)(x,x),\ (T_A \cup T_B)(x,y)\},\\ \min\{(T_A \cup T_B)(x,y),\ (T_A \cup T_B)(y,y)\},\\ \min\{(T_A \cup T_B)(x,z),\ (T_A \cup T_B)(z,z)\}\end{array}\right\}$$

$$=\sup\{\min\{1, 0.3\},\ \min\{0.3, 1\},\ \min\{0.6, 0.5\}\}$$

$$=\sup\{0.3,\ 0.3,\ 0.5\}$$

$$=0.5 \geq 0.4 = \max\{T_A(x,y),\ T_B(x,y)\}$$

$$= (T_A \cup T_B)(x,y).$$

Therefore, union of two DFS equivalence relationsis not DFS equivalence relation on X.

3.14 Definition: A DFS 'R' on a groupoid S is said to be,

(i) Double-framed fuzzy left compatible if $T_R(x, y) \leq T_R(zx, zy)$ and $F_R(x, y) \geq F_R(zx, zy)$ for any x, y, z∈S

(ii) Double-framed fuzzy right compatible if $T_R(x, y) \leq T_R(zx, zy)$ and $F_R(x, y) \geq F_R(zx, zy)$ for any x, y, z ∈ S

(iii) Double-framed fuzzy compatible if $T_R(x,y) \wedge T_R(z, t) \leq T_R(zx, yt)$ and $F_R(x,y) \vee F_R(z, t) \geq F_R(zx, yt)$ for any x, y, z ∈S

3.15 Definition: A DFER on a groupoid 'S' is called,

(i) Double-framed fuzzy left congruency (in short DFLC) if it is double-framed fuzzy left compatible.

(ii) Double-framed fuzzy right congruency (in short DFRC) if it is double-framed fuzzy right compatible.

(iii) Double-framed fuzzy congruency (in short DFFC) if it is double-framed fuzzy compatible.

3.16Proposition:Let R and θ be double-framed fuzzy compatible relationson a groupoid 'S'. Then θoR is also double-framed fuzzy compatible relation on 'S'.

3.17Proposition : Let R and θ be double-framed fuzzy compatible on a groupoid 'S'. Then the full conditions are verified.

(i) θoR ∈DFC(S) (res DFLCS and DFRCS).

(ii) θoR ∈DF(S).

(iii) θoR is a Double-framed fuzzy symmetric.

(iv) θoR = Roθ.

3.18 Proposition: Let 'S' be a semi group and let θ, R ∈DFC(S) of Roθ = θoR, then Roθ ∈ DFC(S).

3.19 Theorem: Let 'S' be a semi group (ie., clear that Δ ∈ DFC(S). Then (DFC(S), Λ, V) is a compatible lattice. We write Δ and ▽as the cost and treated elements if DFC (S).

Let R be adouble-framed fuzzy congruency on a semi group 'S' and let a∈ S. Then DFS R_a in 'S' is called adouble-framed fuzzy congruency class if 'R' containing a∈ S.We will denote the set of double-framed fuzzy congruency classes of R as S/R.

3.20 Proposition : Let 'S' be a regular semi group and let R∈ DFC(S). If R_a is an idempotent element of S/R, then there exists an idempotent e∈ S such that $R_e = R_a$.

CONCLUSION: An attempt has been made to study level cut of double-framed fuzzy soft equivalence relation as well as its some properties such as equivalence class, union and intersection of equivalence relation in this article. Some extension of intuitionistic fuzzy soft relations has been made into double-framed fuzzy soft relations. It is found that intersection of DFSE relations is also DFSE relation, whereas union is not. One can obtain the similar results in pythagorean fuzzy sets and fermatean fuzzy soft sets under suitable norms.

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